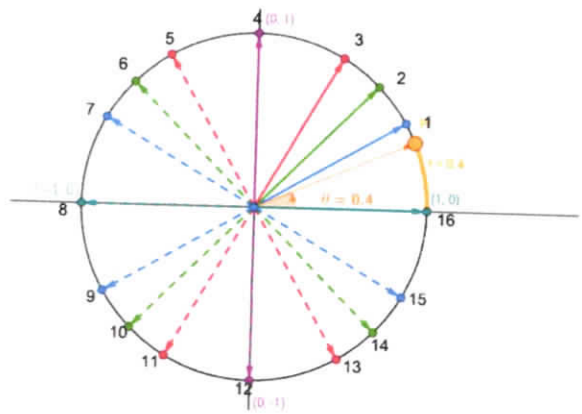
SAMPLE TEST

NOTE: Sample tests are not meant to be a complete study guide. This is just a test that was given on this material one time. Yours will be similar in length and difficulty but will not be exactly the same. There may be topics from the homework that are not covered on this test but WILL be on your test. Working these problems, without referring to notes or solutions should be only ONE PART of your study.

- **Notebook should be turned in before test. It will not be accepted after.**
- **Phones must be turned OFF and put away. Any visible phone (smart watch, headphones, ipad etc.) will result in a grade F .**
- **No scratch paper or notes.**
- **No graphing calculator.**
- **No credit will be given for solutions if work is not shown.**
- **I expect clear and legible presentations .**

(1) Same figure as on homework, see board for colors.

The "blue angles" all have a reference angle of 30 degrees or $\pi/6$ radians.
The "green angles" all have a reference angle of 45 degrees or $\pi/4$ radians.
The "red angles" all have a reference angle of 60 degrees or $\pi/3$ radians.
(ignore the orange here) (12 points)



Write the corresponding number for each of the following angles:

150° 7	$7\pi/6$ 9	210° 9
$7\pi/4$ 14	330° 15	$11\pi/6$ 15
$4\pi/3$ 11	$3\pi/2$ 12	$-\pi/3$ 13
-330° 1	$2\pi/3$ 5	5π 8

What are the coordinates of the points at: 3 points

1) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

2) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

3) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(2) Solve using any of the methods discussed in class. (10 points)

elim y

$$\begin{cases} 2x - y + z = 4 \\ x + 3y + 2z = -1 \\ 7x \quad + 5z = 11 \end{cases}$$

I'm going to eliminate y since it is already gone in eqn 3.

$7x + 5z = 11 \longrightarrow$ elim y $\Rightarrow 7x+5z=11$

3EQ1 $6x - 3y + 3z = 12$

EQ2 $x + 3y + 2z = -1$

add $7x + 5z = 11$

$$\begin{cases} 7x+5z=11 \\ 7x+5z=11 \end{cases}$$

subtract $0=0$

dependent case

Solve for all variable in terms of one (many answers)

let $z=t$, then

$x = \frac{11-5t}{7}$

$2x-y+z=4$

$\Rightarrow y = 2x+z-4 = 2\left(\frac{11-5t}{7}\right)+t-4$

$= -\frac{6}{7} - \frac{3}{7}t$

can check by putting into system

$\boxed{\text{Ans: } \left(\frac{11}{7} - \frac{5}{7}t,\ -\frac{6}{7} - \frac{3}{7}t,\ t\right)}$

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(3) Use Cramer's Rule to solve the following system. (8 points)

$$\begin{cases} x + 3y + z = 2 \\ x + y + 2z = 1 \\ 2x + 3y + 4z = 3 \end{cases}$$

(No credit given for a different method)

$$D = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{vmatrix} = -1$$

$$D_x = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 1 & 2 \\ 3 & 3 & 4 \end{vmatrix} = 2 \qquad x = \frac{D_x}{D} = \frac{2}{-1} = -2$$

$$D_y = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{vmatrix} = -1 \qquad y = \frac{D_y}{D} = \frac{-1}{-1} = 1$$

$$D_z = \begin{vmatrix} 1 & 3 & 2 \\ 1 & 1 & 1 \\ 2 & 3 & 3 \end{vmatrix} = -1 \qquad z = \frac{D_z}{D} = \frac{-1}{-1} = 1$$

$\Rightarrow (-2, 1, 1)$

Can check in system

(4) Given the following matrices: (a-d, 2 points each; e,f 4 points each)

$$A = \begin{bmatrix} 8 & 3 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & -3 & 2 \\ 0 & 5 & 1 & -1 \\ 0 & 2 & 1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 9 & 3 \\ -4 & 1 \end{bmatrix} \quad D = \begin{bmatrix} -1 & 7 & -1 \\ 3 & -2 & 1 \end{bmatrix}$$

Find the following, if possible. (If not possible, say so.)

(a) A + C $= \begin{bmatrix} 17 & 6 \\ -5 & -1 \end{bmatrix}$

(b) AC $= \begin{bmatrix} 60 & 27 \\ -1 & -5 \end{bmatrix}$

(c) BC not possible

(d) det(C)

$$\begin{vmatrix} 9 & 3 \\ -4 & 1 \end{vmatrix} = 9-{}^{-}12 = 21$$

Remember notation

Brackets here straight Bars here.

(e) AD

$$\begin{bmatrix} 8 & 3 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 7 & -1 \\ 3 & -2 & 1 \end{bmatrix}$$

2x2 2x3

$$= \begin{bmatrix} 1 & 50 & -5 \\ -5 & -3 & -1 \end{bmatrix}$$

(f) det (B)

using column 1 since it has two zeros

$$1 \begin{vmatrix} 1 & -3 & 2 \\ 5 & 1 & -1 \\ 2 & 1 & 1 \end{vmatrix} + (-1)3 \begin{vmatrix} -2 & 0 & 4 \\ 5 & 1 & -1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$1(\ 29\ ) - 3\ (\ 8\ ) = 5$$

(5) Given $A = \begin{bmatrix} 0 & -1 & \frac{1}{2} \\ 3 & -4 & 3 \\ 1 & -2 & 3 \end{bmatrix}$

Remember tip... check as you go

(a) Find A^{-1}

(10 points)

$$[A|I] = \left[\begin{array}{ccc|ccc} 0 & -1 & 1/2 & 1 & 0 & 0 \\ 3 & -4 & 3 & 0 & 1 & 0 \\ 1 & -2 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-3R3+R2 \to R2]{R1 \Leftrightarrow R3} \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 0 & 0 & 1 \\ 0 & 2 & -6 & 0 & 1 & -3 \\ 0 & -1 & 1/2 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} -R3 \\ R3 \Leftrightarrow R2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 0 & 0 & 1 \\ 0 & 1 & -1/2 & -1 & 0 & 0 \\ 0 & 2 & -6 & 0 & 1 & -3 \end{array}\right] \xrightarrow[+2R_2+R_1 \to R_1]{-2R_2+R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -2 & 0 & 1 \\ 0 & 1 & -1/2 & -1 & 0 & 0 \\ 0 & 0 & -5 & 2 & 1 & -3 \end{array}\right] -\frac{1}{5}R_3 \to R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & -2 & 0 & 1 \\ 0 & 1 & -1/2 & -1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{2}{5} & -\frac{1}{5} & \frac{3}{5} \end{array}\right] \xrightarrow[1/2R_3+R_2 \to R_2]{-2R_3+R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -6/5 & 2/5 & -1/5 \\ 0 & 1 & 0 & -6/5 & -1/10 & +3/10 \\ 0 & 0 & 1 & -2/5 & -1/5 & 3/5 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -6/5 & 2/5 & -1/5 \\ -6/5 & -1/10 & 3/10 \\ -2/5 & -\frac{1}{5} & 3/5 \end{bmatrix}$$

can check $AA^{-1} = A^{-1}A = I$

(b) Solve the system of equations by writing it as a matrix equation Ax=B and using the inverse of the coefficient matrix (which you found in part a).

$$\begin{cases} -y + \frac{1}{2}z = 7 \\ 3x - 4y + 3z = 1 \\ x - 2y + 3z = 2 \end{cases}$$

(3 points)

$$\begin{bmatrix} 0 & -1 & 1/2 \\ 3 & -4 & 3 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix}$$

$$A \quad X = B$$

solution is

$$X = A^{-1}B = \begin{bmatrix} -6/5 & 2/5 & -1/5 \\ -6/5 & -1/10 & 3/10 \\ -2/5 & -1/5 & 3/5 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{42}{5} \\ -\frac{79}{10} \\ -\frac{9}{5} \end{bmatrix}$$

$$\left(-\frac{42}{5}, -\frac{79}{10}, -\frac{9}{5}\right)$$

can check in system

(6) (a) Convert from DMS (degree, minute seconds)to decimal degrees, show work. $19°45'72''$

(8 points)

$45' \cdot \frac{1°}{60'} = \frac{45}{60} = \frac{3}{4} = .75°$ $\quad 72'' \cdot \frac{1°}{3600''} = \frac{2}{100} = 0.02 \Rightarrow 19.77°$

(b) Convert from decimal degrees to DMS , show work. $42.6°$

$.6° \cdot \frac{60'}{1°} = \frac{6}{10} \cdot 60 = 36' \Rightarrow 42°36'$

(c) Convert from radians to degrees: $\frac{7\pi}{9}$

$\frac{7\pi}{9} \cdot \frac{180°}{\pi} = 140°$

(d) Convert from degrees to radians, exactly (no calculator): $12°$

$12° \cdot \frac{\pi}{180°} = \frac{\pi}{15}$ radians

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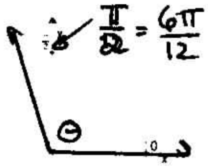
(7) Graph the angle $\theta = 7\pi/12$ in standard position. Give two coterminal angles, one of which is positive and the other negative. Find the reference angle. (8 points)

$\frac{\pi}{2} = \frac{6\pi}{12}$ $\quad \frac{7\pi}{12}$ is just past $\frac{6\pi}{12}$



$\frac{12\pi}{12} = \pi$

Coterminal positve $\underline{\frac{31\pi}{12}}$ $\quad$ Coterminal negative $\underline{\frac{-17\pi}{12}}$ $\quad$ Ref angle $\underline{\frac{5\pi}{12}}$

$\frac{7\pi}{12} + 2\pi$ $\quad \frac{7\pi}{12} - 2\pi$ $\quad \pi - \frac{7\pi}{12}$

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(8) (For each of the following acute angles, find 4 angles, one in each quadrant, having the given angle as a reference angle. Answer in the units given, exactly. (12 points)

	Q1	Q2	Q3	Q4
$23°$	$23°$	$180° - 23° = 157°$	$180° + 23° = 203°$	$360° - 23° = 337°$
$2\pi/5$	$2\pi/5$	$\pi - \frac{2\pi}{5} = 3\pi/5$	$\pi + \frac{2\pi}{5} = \frac{7\pi}{5}$	$2\pi - \frac{2\pi}{5} = 8\pi/5$
0.2	0.2	$\pi - 0.2$	$\pi + 0.2$	$2\pi - 0.2$

(7) Use matrix methods (Gaussian elimination or Gauss Jordan) to solve: (10 points)

$$\begin{cases} -x-2y-z=-3 \\ 2x+y+z=16 \\ x+y+2z=9 \end{cases}$$

You must obtain row echelon form or reduced row echelon form. Be sure to label operations performed at each step.

$$\left[\begin{array}{ccc|c} -1 & -2 & -1 & -3 \\ 2 & 1 & 1 & 16 \\ 1 & 1 & 2 & 9 \end{array}\right] \xrightarrow[R_1+R_3\to R_3]{\substack{-R_1\to R_1 \\ 2R_1+R_2\to R_2}} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -3 & -1 & 10 \\ 0 & -1 & 1 & 6 \end{array}\right] \xrightarrow{\substack{-R_3\to R_3 \\ R_2\leftrightarrow R_3}}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -6 \\ 0 & -3 & -1 & 10 \end{bmatrix} \xrightarrow{3R_2+R_3\to R_3} \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & -4 & -8 \end{bmatrix} \xrightarrow{-\frac{1}{4}R_3\to R_3} \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

This is row echelon form. We can write corresp. system & solve by back subst. or keep goin's

$R_3+R_1\to R_1$

$R_3+R_2\to R_2$

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{-2R_2+R_1\to R_1} \begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Soln: $(9,-4,2)$

can check